

Near Real-Time Airborne Total Propagated Uncertainty Framework Bathy Lidar



Presented by:
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Overall Classification: **UNCLASSIFIED**

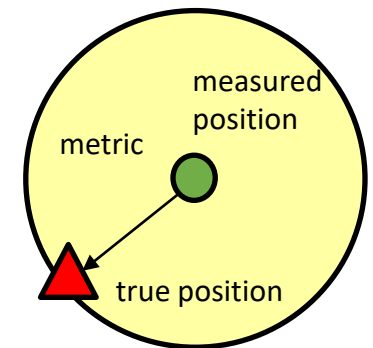
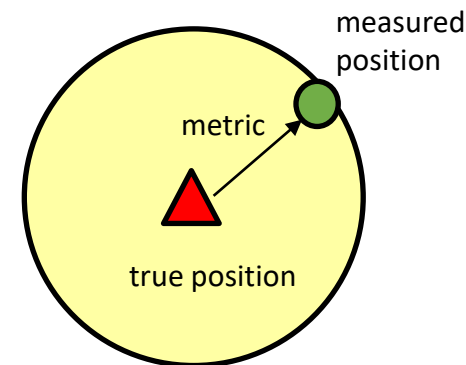
Outline

- Why Total Propagate Uncertainty?
- Some History
- Calculation Framework
- Modeling and Simulation

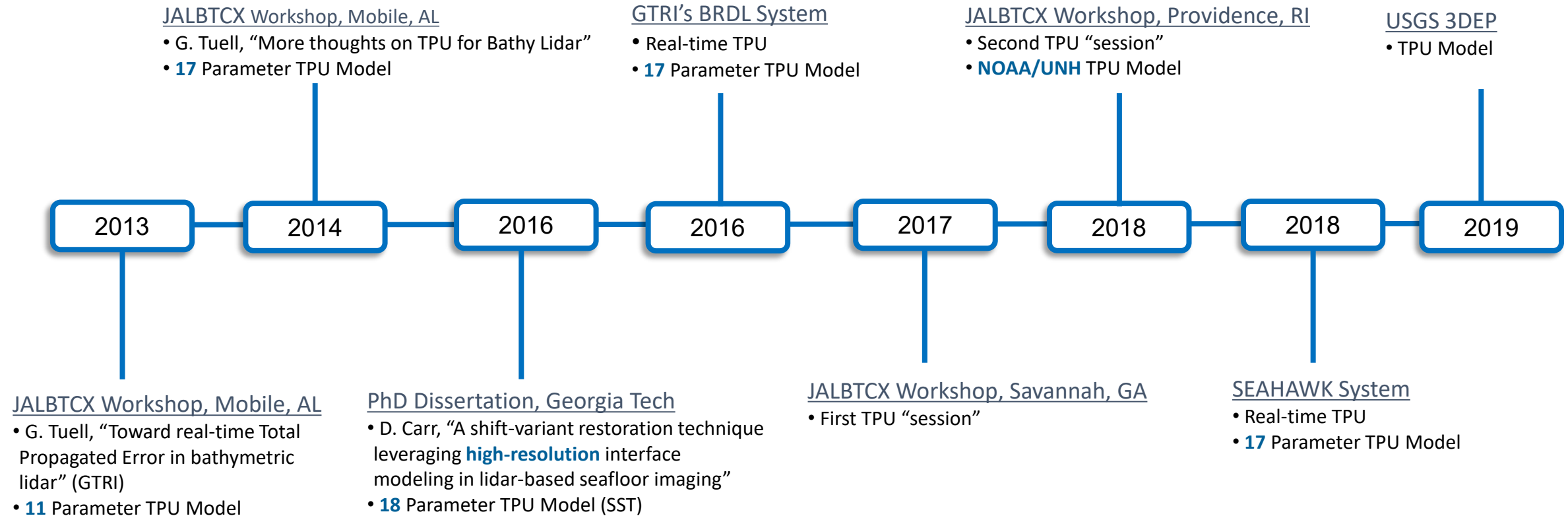


Why Uncertainty?

- Any **dataset** is only a representation of reality, therefore contains **uncertainty** about the nature of the real world being represented.
(De, Goodchild and Longley, 2007)
- Bathy Lidar Sources of Uncertainty
 - Trajectory, boresight, sensor measurements, water characteristics and ranging values with variances.
 - Total Propagated Uncertainty (TPU)
- How much is acceptable?
 - Surveying / Mapping
 - Specification / Standard



Recent TPU History



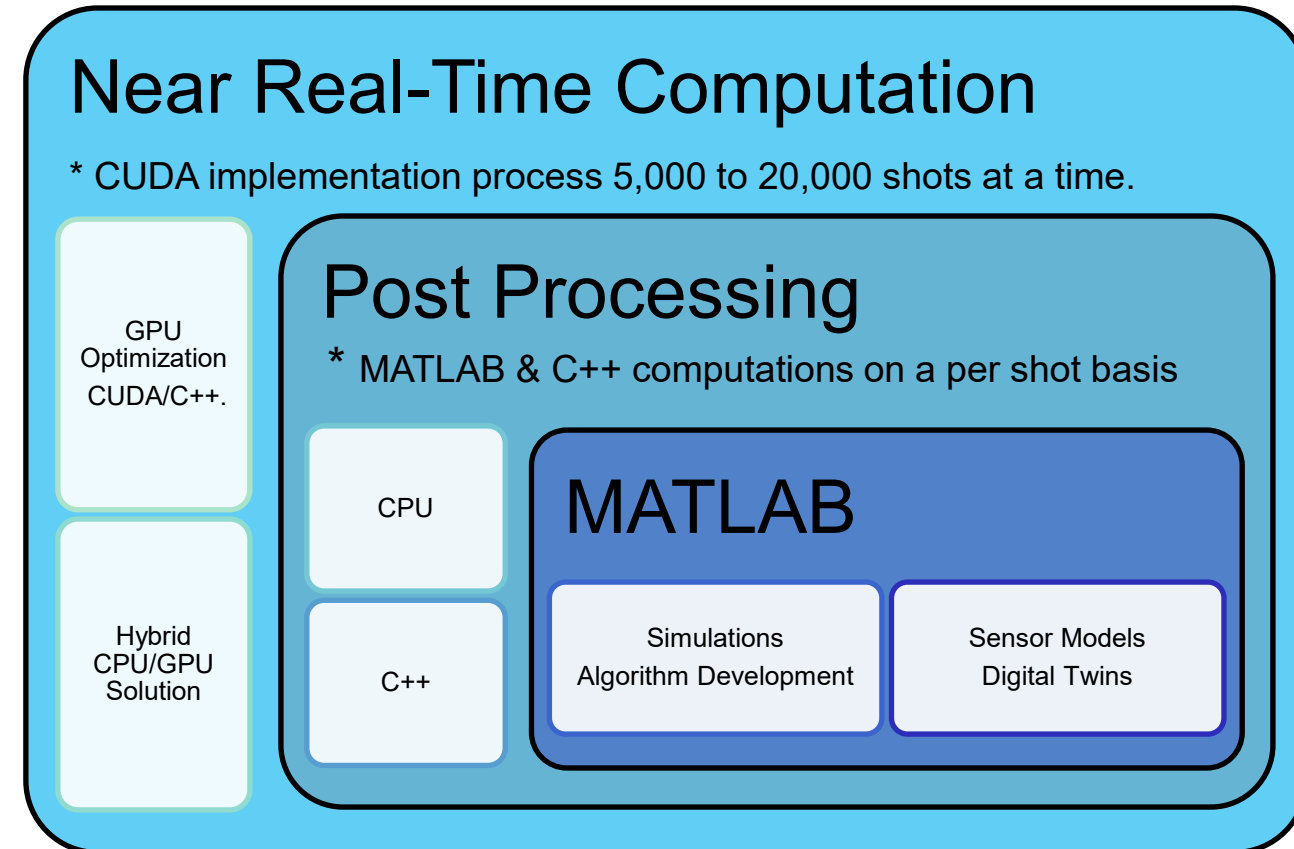
$$\begin{bmatrix} N_P \\ E_P \\ D_P \end{bmatrix} = R_Z(\kappa)R_Y(\varphi)R_X(\omega)R_Z(\theta) \left[R_Y(\phi_A) \begin{bmatrix} 0 \\ 0 \\ l_A \end{bmatrix} + R_Y(\phi_W) \begin{bmatrix} 0 \\ 0 \\ l_W \end{bmatrix} \right] + \begin{bmatrix} N_T \\ E_T \\ D_T \end{bmatrix}$$

$$\begin{bmatrix} N_P \\ E_P \\ D_P \end{bmatrix} = R_z(k)R_Y(\varphi)R_x(\omega) \left(R_z(\Delta k)R_Y(\Delta\varphi)R_x(\Delta\omega)R_z(\theta) \left[R_Y(\phi_a) \begin{bmatrix} 0 \\ 0 \\ l_A \end{bmatrix} + R_Y(\phi_w) \begin{bmatrix} 0 \\ 0 \\ l_w \end{bmatrix} \right] + \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} \right) + \begin{bmatrix} N_T \\ E_T \\ D_T \end{bmatrix}$$

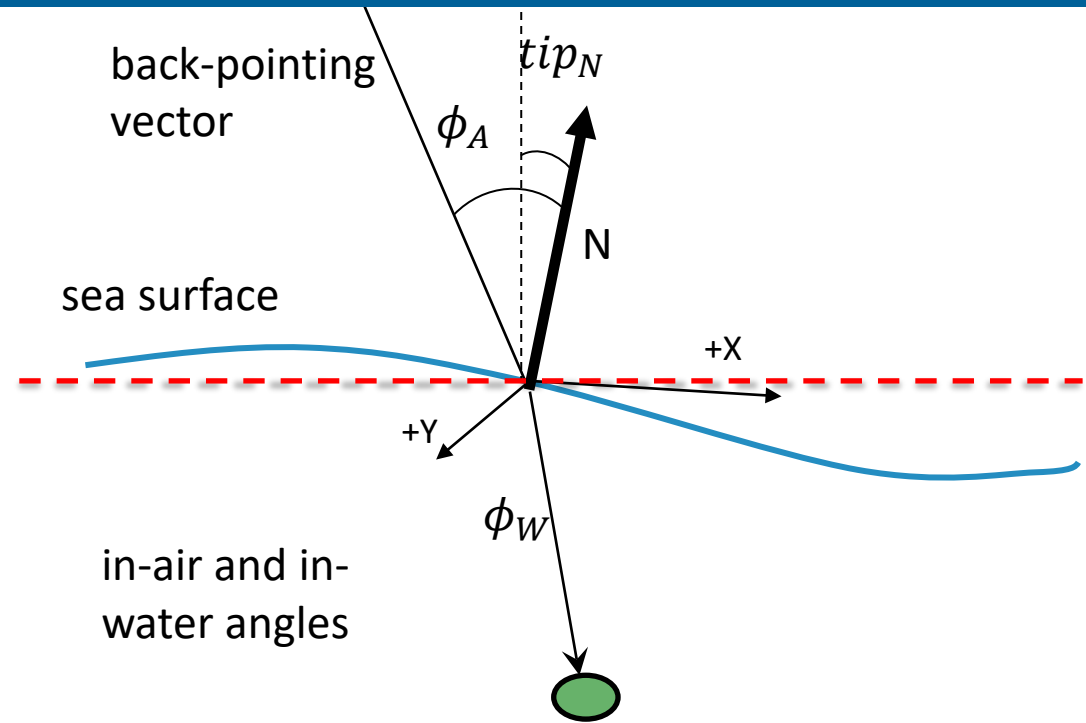
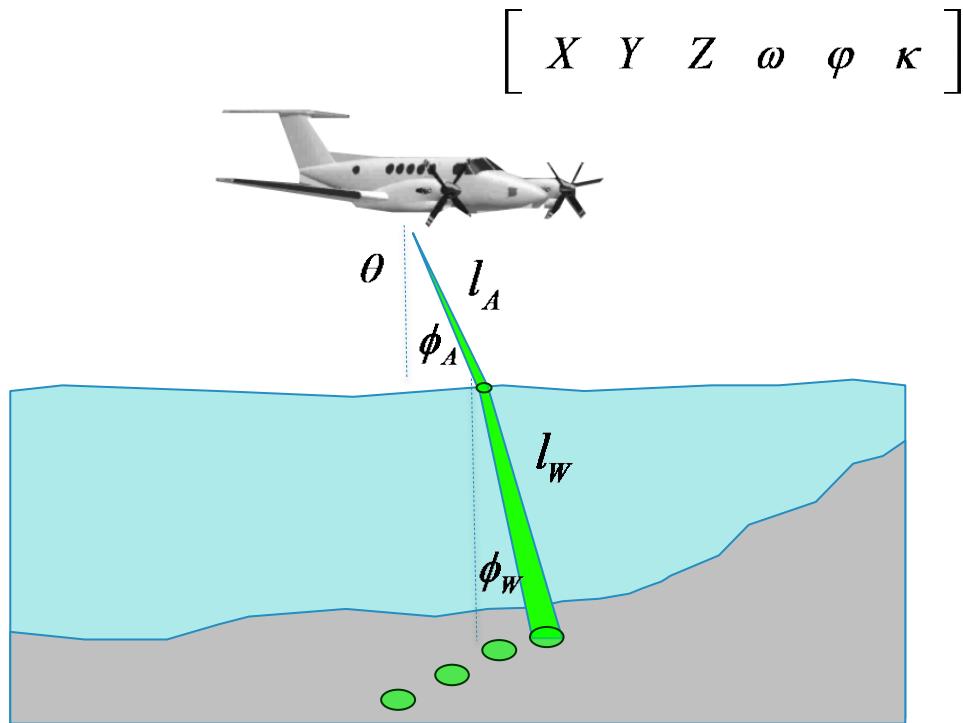


TPU Computational Framework

- Nested Approach
 - MATLAB
 - Implementation in C++ /CPU
 - GPU optimization in CUDA/C++
 - **Compute Unified Device Architecture (CUDA)** is a new approach to solve complex problems by transforming the GPU into a massive parallel processor
 - **140,000** TPU values per second
 - **Hybrid** CPU/GPU solution create a live point cloud with TPU in real time



Traditionally ...TPU is computed using 17-parameter model



surface often assumed to be horizontally flat

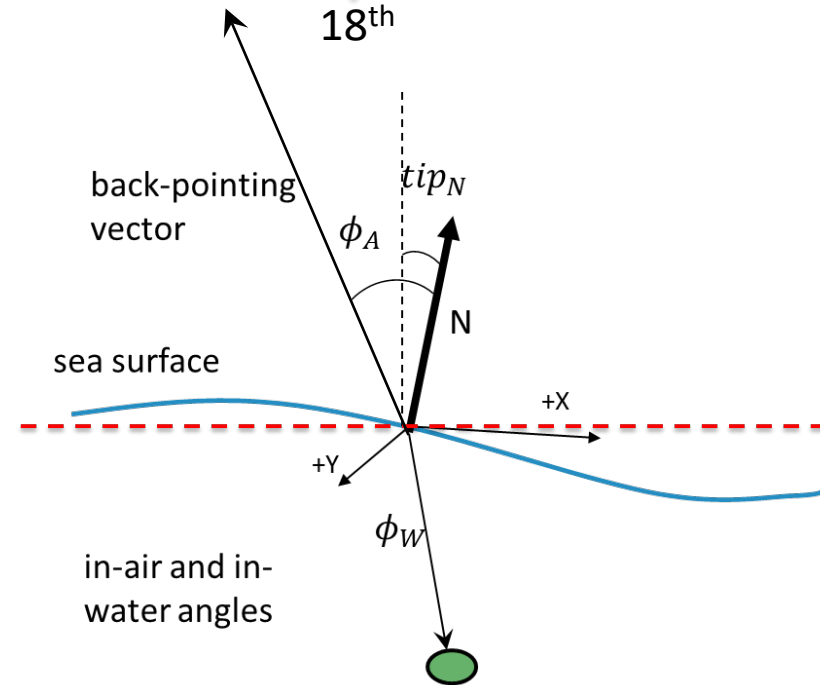
$$\begin{bmatrix} N_P \\ E_P \\ D_P \end{bmatrix} = R_z(k)R_Y(\phi)R_x(\omega) \left(R_z(\Delta k)R_Y(\Delta\phi)R_x(\Delta\omega)R_z(\theta) \left(R_Y(\phi_a) \begin{bmatrix} 0 \\ 0 \\ l_A \end{bmatrix} + R_Y(\phi_w) \begin{bmatrix} 0 \\ 0 \\ l_w \end{bmatrix} + \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} \right) + \begin{bmatrix} N_T \\ E_T \\ D_T \end{bmatrix} \right)$$

18 Parameter TPU Model

- For every seafloor coordinate

$$\begin{bmatrix} E \\ N \\ U \end{bmatrix}_M = R_Z(-h)R_x(p)R_y(r) \left[R_Z(-\Delta h)R_x(\Delta p)R_y(\Delta r)R_Z(-\theta - \Delta\theta) \left(R_x(\phi_A) \begin{bmatrix} 0 \\ 0 \\ -l_A \end{bmatrix} + R_x(\phi_W)R_y(\omega_W) \begin{bmatrix} 0 \\ 0 \\ -l_W \end{bmatrix} \right) + \begin{bmatrix} \Delta Y \\ \Delta X \\ -\Delta Z \end{bmatrix} \right] + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

- 18 Parameters
 - Trajectory
 - Boresight
 - Sensor measurements
 - Water characteristics
 - Ranging values with variances
 - Sea Surface Topography**



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TPU Computation

- A series of matrix operations compute the partial and inner derivatives

$\sigma_{N_T}^2$	$\sigma_{N_T E_T}$	$\sigma_{N_T D_T}$	$\sigma_{N_T k}$	$\sigma_{N_T \varphi}$	$\sigma_{N_T \omega}$	$\sigma_{N_T \Delta k}$	$\sigma_{N_T \Delta \varphi}$	$\sigma_{N_T \Delta \omega}$	$\sigma_{N_T \Delta X}$	$\sigma_{N_T \Delta Y}$	$\sigma_{N_T \Delta Z}$	$\sigma_{N_T \theta}$	$\sigma_{N_T \phi_A}$	$\sigma_{N_T l_A}$	$\sigma_{N_T l_w}$	$\sigma_{N_T \phi_w}$	$\sigma_{N_T \omega_w}$
$\cdot$	$\sigma_{E_T}^2$	$\sigma_{E_T D_T}$	$\sigma_{E_T k}$	$\sigma_{E_T \varphi}$	$\sigma_{E_T \omega}$	$\sigma_{E_T \Delta k}$	$\sigma_{E_T \Delta \varphi}$	$\sigma_{E_T \Delta \omega}$	$\sigma_{E_T \Delta X}$	$\sigma_{E_T \Delta Y}$	$\sigma_{E_T \Delta Z}$	$\sigma_{E_T \theta}$	$\sigma_{E_T \phi_A}$	$\sigma_{E_T l_A}$	$\sigma_{E_T l_w}$	$\sigma_{E_T \phi_w}$	$\sigma_{E_T \omega_w}$
$\cdot$	$\cdot$	$\sigma_{D_T}^2$	$\sigma_{D_T k}$	$\sigma_{D_T \varphi}$	$\sigma_{D_T \omega}$	$\sigma_{D_T \Delta k}$	$\sigma_{D_T \Delta \varphi}$	$\sigma_{D_T \Delta \omega}$	$\sigma_{D_T \Delta X}$	$\sigma_{D_T \Delta Y}$	$\sigma_{D_T \Delta Z}$	$\sigma_{D_T \theta}$	$\sigma_{D_T \phi_A}$	$\sigma_{D_T l_A}$	$\sigma_{D_T l_w}$	$\sigma_{D_T \phi_w}$	$\sigma_{D_T \omega_w}$
$\cdot$	$\cdot$	$\cdot$	σ_k^2	$\sigma_k \varphi$	$\sigma_k \omega$	$\sigma_k \Delta k$	$\sigma_k \Delta \varphi$	$\sigma_k \Delta \omega$	$\sigma_k \Delta X$	$\sigma_k \Delta Y$	$\sigma_k \Delta Z$	$\sigma_k \theta$	$\sigma_k \phi_A$	$\sigma_k l_A$	$\sigma_k l_w$	$\sigma_k \phi_w$	$\sigma_k \omega_w$
$\cdot$	$\cdot$	$\cdot$	$\cdot$	σ_φ^2	$\sigma_\varphi \omega$	$\sigma_\varphi \Delta k$	$\sigma_\varphi \Delta \varphi$	$\sigma_\varphi \Delta \omega$	$\sigma_\varphi \Delta X$	$\sigma_\varphi \Delta Y$	$\sigma_\varphi \Delta Z$	$\sigma_\varphi \theta$	$\sigma_\varphi \phi_A$	$\sigma_\varphi l_A$	$\sigma_\varphi l_w$	$\sigma_\varphi \phi_w$	$\sigma_\varphi \omega_w$
$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	σ_ω^2	$\sigma_\omega \Delta k$	$\sigma_\omega \Delta \varphi$	$\sigma_\omega \Delta \omega$	$\sigma_\omega \Delta X$	$\sigma_\omega \Delta Y$	$\sigma_\omega \Delta Z$	$\sigma_\omega \theta$	$\sigma_\omega \phi_A$	$\sigma_\omega l_A$	$\sigma_\omega l_w$	$\sigma_\omega \phi_w$	$\sigma_\omega \omega_w$
$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\sigma_{\Delta k}^2$	$\sigma_{\Delta k \Delta \varphi}$	$\sigma_{\Delta k \Delta \omega}$	$\sigma_{\Delta k \Delta X}$	$\sigma_{\Delta k \Delta Y}$	$\sigma_{\Delta k \Delta Z}$	$\sigma_{\Delta k \theta}$	$\sigma_{\Delta k \phi_A}$	$\sigma_{\Delta k l_A}$	$\sigma_{\Delta k l_w}$	$\sigma_{\Delta k \phi_w}$	$\sigma_{\Delta k \omega_w}$
$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\sigma_{\Delta \varphi}^2$	$\sigma_{\Delta \varphi \Delta \omega}$	$\sigma_{\Delta \varphi \Delta X}$	$\sigma_{\Delta \varphi \Delta Y}$	$\sigma_{\Delta \varphi \Delta Z}$	$\sigma_{\Delta \varphi \theta}$	$\sigma_{\Delta \varphi \phi_A}$	$\sigma_{\Delta \varphi l_A}$	$\sigma_{\Delta \varphi l_w}$	$\sigma_{\Delta \varphi \phi_w}$	$\sigma_{\Delta \varphi \omega_w}$
$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\sigma_{\Delta \omega}^2$	$\sigma_{\Delta \omega \Delta X}$	$\sigma_{\Delta \omega \Delta Y}$	$\sigma_{\Delta \omega \Delta Z}$	$\sigma_{\Delta \omega \theta}$	$\sigma_{\Delta \omega \phi_A}$	$\sigma_{\Delta \omega l_A}$	$\sigma_{\Delta \omega l_w}$	$\sigma_{\Delta \omega \phi_w}$	$\sigma_{\Delta \omega \omega_w}$
$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\sigma_{\Delta X}^2$	$\sigma_{\Delta X \Delta Y}$	$\sigma_{\Delta X \Delta Z}$	$\sigma_{\Delta X \theta}$	$\sigma_{\Delta X \phi_A}$	$\sigma_{\Delta X l_A}$	$\sigma_{\Delta X l_w}$	$\sigma_{\Delta X \phi_w}$	$\sigma_{\Delta X \omega_w}$
$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\sigma_{\Delta Y}^2$	$\sigma_{\Delta Y \Delta Z}$	$\sigma_{\Delta Y \theta}$	$\sigma_{\Delta Y \phi_A}$	$\sigma_{\Delta Y l_A}$	$\sigma_{\Delta Y l_w}$	$\sigma_{\Delta Y \phi_w}$	$\sigma_{\Delta Y \omega_w}$
$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\sigma_{\Delta Z}^2$	$\sigma_{\Delta Z \theta}$	$\sigma_{\Delta Z \phi_A}$	$\sigma_{\Delta Z l_A}$	$\sigma_{\Delta Z l_w}$	$\sigma_{\Delta Z \phi_w}$	$\sigma_{\Delta Z \omega_w}$
$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	σ_θ^2	$\sigma_\theta \phi_A$	$\sigma_\theta l_A$	$\sigma_\theta l_w$	$\sigma_\theta \phi_w$	$\sigma_\theta \omega_w$
$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\sigma_{\phi_A}^2$	$\sigma_{\phi_A l_A}$	$\sigma_{\phi_A l_w}$	$\sigma_{\phi_A \phi_w}$	$\sigma_{\phi_A \omega_w}$
$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\sigma_{l_A}^2$	$\sigma_{l_A l_w}$	$\sigma_{l_A \phi_w}$	$\sigma_{l_A \omega_w}$
$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\sigma_{l_w}^2$	$\sigma_{l_w \phi_w}$	$\sigma_{l_w \omega_w}$
$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\sigma_{\phi_w}^2$	$\sigma_{\phi_w \omega_w}$
$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\sigma_{\omega_w}^2$

$$\tilde{\Sigma}_M(1:16;1:16) = \Sigma_M(1:16;1:16)$$

From Tuell (2014)

Navigation

Pointing Ranging Performance
Calibration

Sea surface:
2 variables

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TPU Computation

- 3x3 matrix of variance – covariance

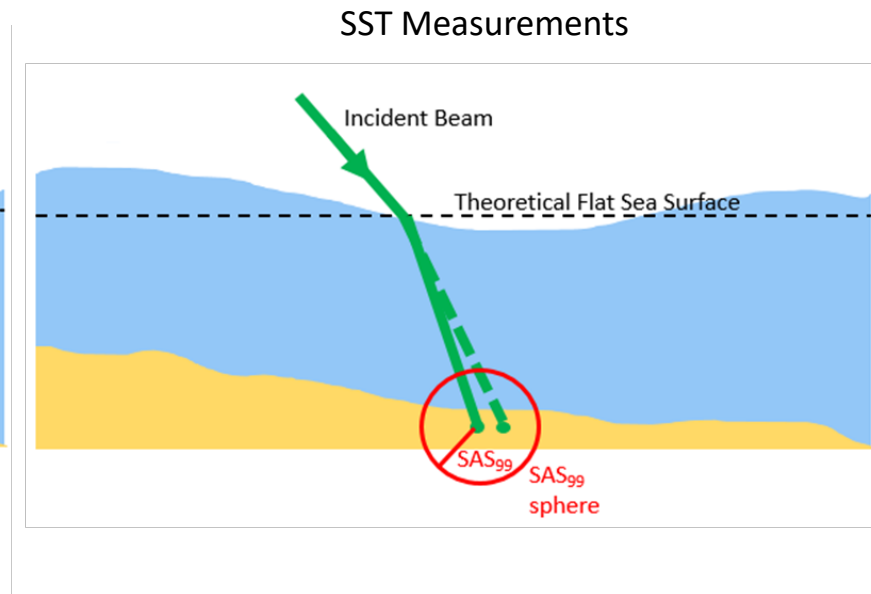
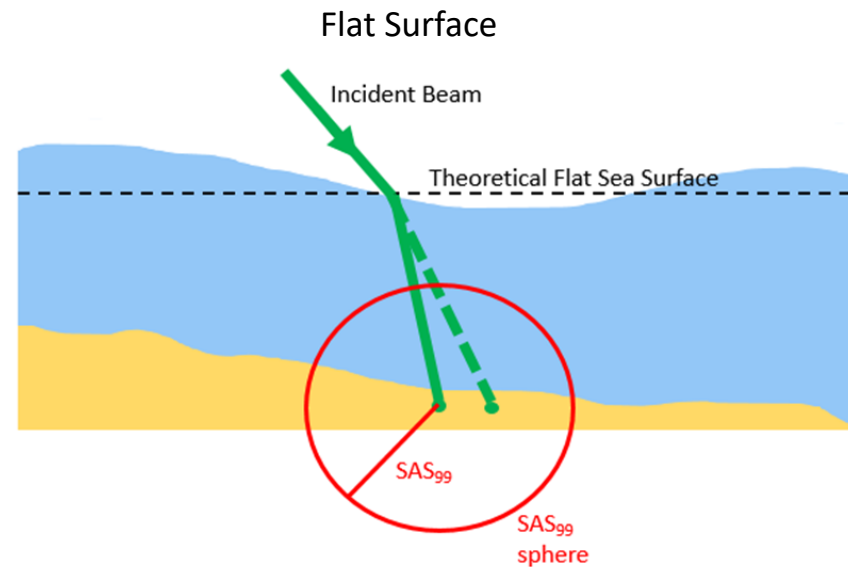
$$\Sigma_V = \begin{bmatrix} \sigma_E^2 & \sigma_{EN} & \sigma_{EU} \\ \sigma_{NE} & \sigma_N^2 & \sigma_{NU} \\ \sigma_{UE} & \sigma_{UN} & \sigma_U^2 \end{bmatrix} = J \Sigma_M J^T$$

3 variances representing ENU
6 representing off-axis components

- Single value representing accuracy

$$SAS_{99} = 1.122(\sigma_N + \sigma_E + \sigma_D)$$

Spherical Accuracy Standard 99%



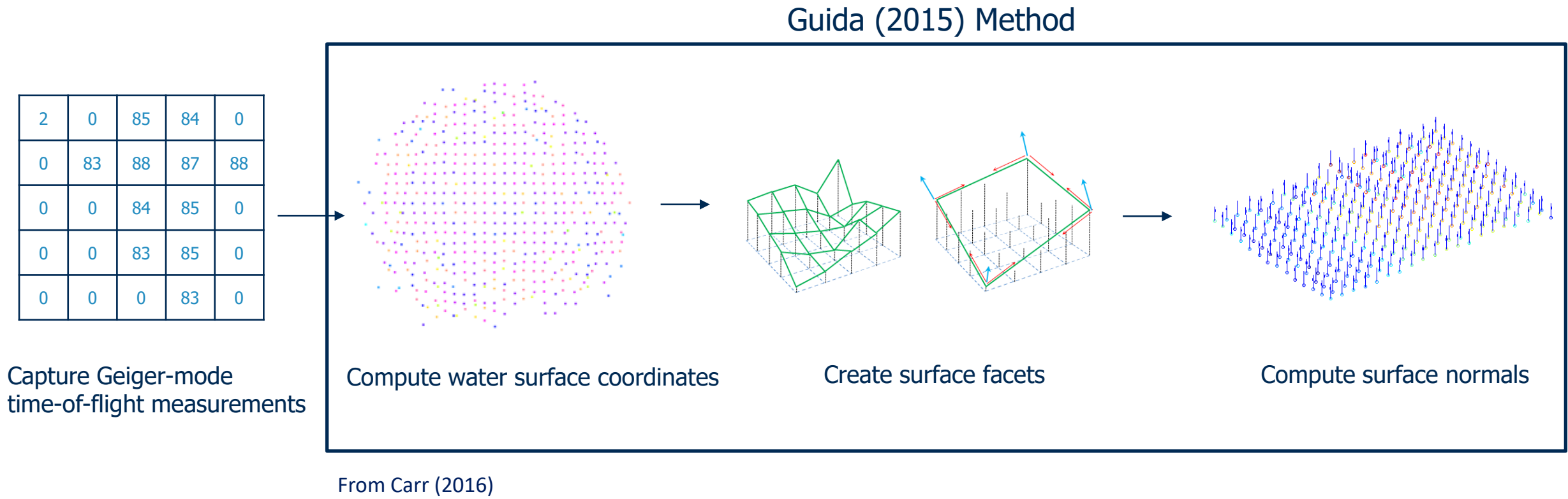
Geo location error and uncertainty are amplified when a flat surface is assumed.

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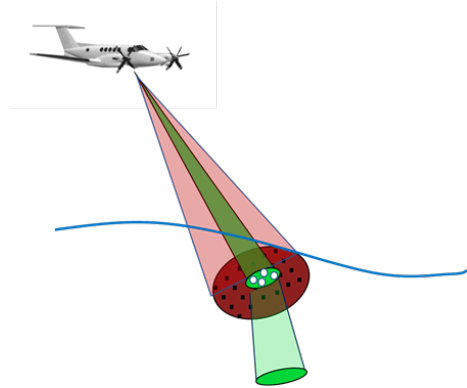


High Resolution SST

- Segmented detectors on Co-located Green and IR.



Development Design and Testing



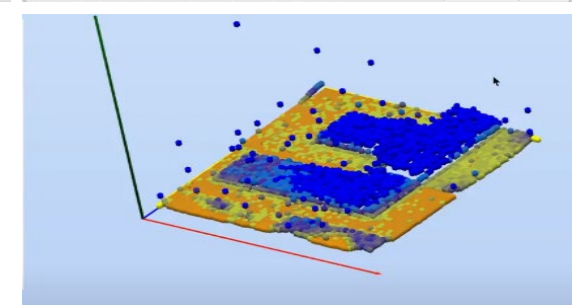
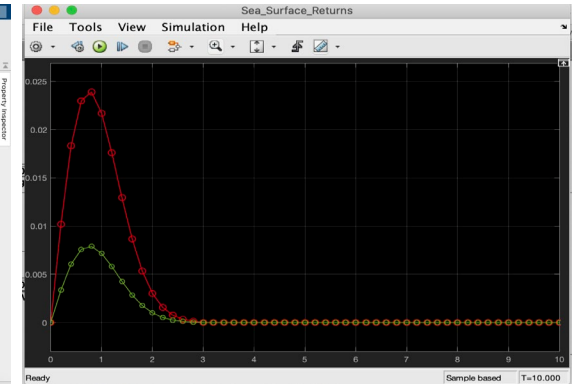
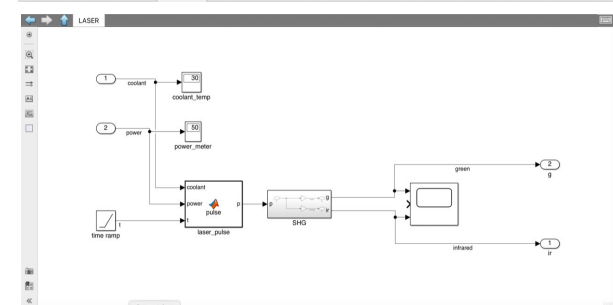
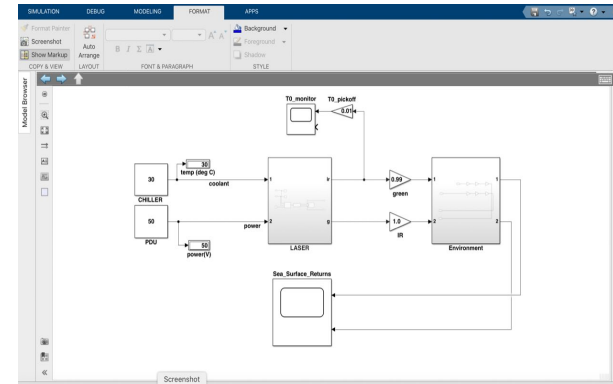
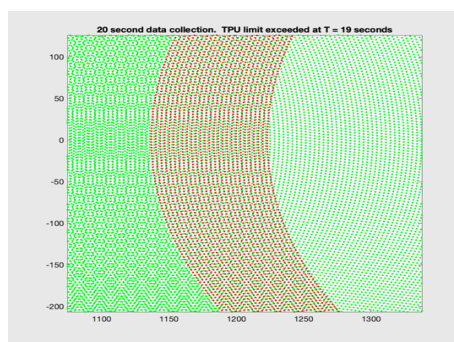
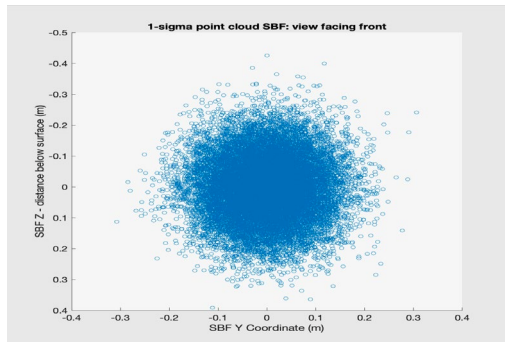
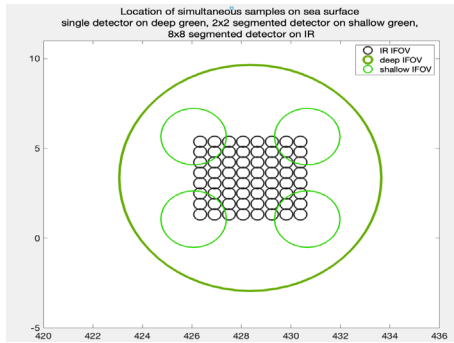
MATLAB + Simulink

Simulations
Algorithm Development

Sensor Models
Digital Twins

```
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bulldog_TPU_ENU_vector_raytrace_sea_surface_20201124.m
/environment, indexwater;

green_natural_averison = 0.25; % seconds ANSI Z136.1 section 4.4.4.2.3.2
%% 5. Enter lidar measurements and their sigmas .....
105
106
107
108
109
110 nA = environment.indexAir; % lidar scan angle in degrees
111 nW = environment.indexWater; % lidar scan angle in degrees
112 pA = system.indexAir; % lidar scan angle in degrees
113 t = 0.0; % optical path length in air in meters
114 tA = 3078.0; % optical path length in air in meters
115 pW = environment.indexWater; % in-water incidence angle in degrees
116 oW = 0.0; % sea surface roll angle in degrees
117 tW = 0.0; % optical path length in water in meters
118
119 sPA = 0.05; % sigma of pA orientation in degrees
120 st = 0.05; % sigma of t orientation in degrees
121 sIA = 0.1; % sigma of tA in meters
122 spW = 5.0; % sigma of pW in degrees
123 sOW = 0.0; % sigma of oW in degrees
124 sIW = 0.1; % sigma of tW in meters
125
126 %% 6. Compute LGF coordinate vector of deep green at sea surface, (V) ....
127
128 A = [cosd(-h) -sind(-h) 0.0; sind(-h) cosd(-h) 0.0; 0.0 0.0 1.0];
129 R = [1 0 0; 0 cosd(n) -sind(n); 0 sind(n) cosd(n)];
```



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SUMMARY

- A GPU accelerated TPU calculation from multiple channels, each creating thousands of waveforms per second is achievable, reliable, and useful.
- Near real-time TPU will be especially beneficial for assessing data quality when performing airborne surveying operations in remote locations.



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- Eric Cahoon

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